A Cohort-Specific Approach to Retirement Savings

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A cohort-specific collar approach to retirement security*

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Abstract: This paper develops a pension product that is explicit about the pre-established goals that are aimed for. The proposed product presents a trade-off that is transparent in terms of required contributions, the income level targeted and guarantees offered. Depending on participants' preferences, individualized investment plans are constructed that focus on achieving the desired standard of living in retirement with a likelihood that is ex-ante defined. In addition, the event of falling short of the goal is managed. To address the risk of failure to achieve the desired income target, guaranteed income is offered to safeguard a minimum standard of living in retirement. Given the contribution rate participants can afford to save, the set of product parameters has to be chosen. Apart from choosing the desired and guaranteed level of income, participants are asked to determine the likelihood of realizing the desired benefit and the likelihood of ending up with the guarantee. The proposed approach is compared to life cycle strategies more commonly executed in the pension industry. The results indicate that the collar approach outperforms the evaluated life cycle strategies in terms of the desired replacement rate that can be offered given the replacement rate in adverse scenarios. Moreover, the approach offers a larger probability of realizing the desired benefit given a predefined level of downside risk.

Keywords: retirement security redesign, life cycle investment, contingent claims analysis, dynamic portfolio selection

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1 Introduction

Worldwide, traditional defined benefit (DB) plans experienced a solvency crisis at the beginning of the twenty-first century. The crisis was triggered by the joint impact of adverse shocks at global financial markets and the introduction of market-based accounting standards. Falling stock market returns and historically low long-dated fixed-income yields caused a severe drop in asset values and a sharp increase in the market value of pension liabilities. Moreover, the maturing of DB plans led to substantial funding pressure. As pension liabilities expanded relative to the premium base, funds experienced a reduced ability to absorb unanticipated shocks in financial markets and longevity. With financial and actuarial risks of the pension liabilities starting to dominate those of the core business, sponsoring companies no longer wanted to underwrite the risks of their pension funds (Boeri, Bovenberg, Cœuré and Roberts, 2006). As a result, many countries experienced a shift from DB plans towards defined contribution (DC) plans. Particularly, in the Anglo-Saxon countries traditional DB plans have largely been replaced by individual employee-managed retirement accounts (Munnell, 2006).

The shift to individual DC is not, however, perceived to be the long-run answer for employer-sponsored retirement plans (Merton, 2006). Although DC plans are well defined in terms of risk exposure for the sponsor, they are too complex and risky for participants. In traditional DC schemes, participants are asked to make a wide range of detailed investment decisions to secure their desired benefit for in retirement. These financial decisions are far more complex that the ones that participants were called to decide upon in the past (Merton, 2006). In the traditional DB model, plan participants were allowed to plan their retirement income without requiring much knowledge about saving or portfolio choice. In fact, academic literature reveals that individuals typically lack the basic financial knowledge and computational ability to implement effective financial life cycle planning (Lusardi and Mitchell, 2007; Van Rooij, Lusardi and Alessie, 2011; Van Rooij, Kool and Prast, 2007). As individuals furthermore tend to delay saving for retirement and often save too little (Thaler and Benartzi, 2004; Kooreman and Prast, 2010), it is not regarded to be the adequate solution to leave the responsibility of securing sufficient retirement income with households.

Given the cost of running DB plans and the complexity associated with individual DC schemes, innovations towards retirement security are necessary to provide an answer to the challenging task of individual life cycle planning. This paper aims at the development of an innovative pension product that produces a secure and simple DB-like payout pattern by implementing a DC-type institutional structure. To produce the promised benefit, a dynamic portfolio strategy is executed to transform a sequence of contribution payments into a benefit linked to a participant's replacement rate of final pay. However, rather than asking participants to implement the investment strategy themselves, portfolio decisions are delegated to the pension plan to solve the dynamic investment problem on behalf of them. Participants are only asked to make a set of choices in order to translate their preferences into investment decisions needed to arrive at their income target. The choices participants are asked to decide on, are framed in questions that are relevant and meaningful to them. For instance, what would be your desired standard of living in retirement and how much can you afford to save?[1]

The proposed pension product is explicit about the pre-established goals that are aimed for. As

[1]Empirical research shows that Dutch employees are readily able to express their desired standard of living in terms of desired spending in retirement (Prast, 2007).

such, the product presents a trade-off that is transparent in terms of required contributions, the income level targeted and guarantees offered. Depending on participants' preferences, individualized investment plans are constructed that focus on realizing the income target with a likelihood that is ex-ante defined. To enhance the likelihood of achieving the desired income in retirement, the pension product requires the acceptance of forgoing the opportunity to obtain a benefit in excess of the desired benefit. Because the value of the upside is used to increase the probability of reaching the goal, the desired benefit represents the maximum attainable benefit. In addition, the event of falling short of the goal is managed. To address the risk of failure to achieve the desired income target, guaranteed income is offered to safeguard a minimum standard of living in retirement. To specify the preferred product, participants have to decide on a set of product parameters. Apart from choosing the desired and guaranteed level of retirement income, participants are asked to determine the probability of realizing the desired benefit and the probability of ending up with the guarantee.

The set of choices regarding the product parameters determines the price of the product. Evidently, appealing choices require large contribution payment over the life cycle. As a consequence, participants face a trade-off between the parameters given the contribution level they can afford to save. To present insight in the set of feasible choices to make, the trade-off between contribution, ambition and guarantees is illustrated in this paper. For instance, if the probability of realizing the desired income target is considered to be too small, one may decide to increase the probability by either selecting a lower desired benefit, a lower guaranteed benefit, or a larger probability of ending up with the guarantee. Alternatively, one could choose to pay more contributions. Once participants have decided on their benefit profile and the contribution rate required to finance this profile, the investment strategy is explicitly defined. The dynamic investment strategy corresponding to the preferred payoff follows from a specific algorithm based on participants' choices. Hence, rather than taking investment decisions, participants are expected to specify their desired product.

Specifically, the pension product is defined by an option contingent on the state of the economy at retirement. As the product offers downside protection in adverse scenarios and limited upside potential in prosperous economic scenarios, the product can be framed in terms of owning a protective put option and writing a covered call option. At financial markets, contingent claims composed of similar option positions are called collar options. The pension product analyzed in this paper is defined by a collar option, the payoff of which aims to provide participants with lifetime income in retirement. As such, the maximum and minimum payoff of the collar option are given by the price of real annuities corresponding to a desired and guaranteed replacement rate of final pay. In order to live up to the targeted benefit, the payoff of the collar option is constructed synthetically by dynamically adjusting portfolio weights as time passes and asset prices move. As our aim is to present a first analysis of the trade-off between contribution, ambition and guarantees, we focus on portfolio choice within the familiar continuous-time complete market setting. Accordingly, the payoff of the claim can be replicated perfectly by implementing a delta replicating investment strategy.

In determining participant's asset allocation, all assets dedicated to arrive at the retirement goals have to be considered. Therefore, the strategy takes into account accumulated savings as well as wealth possessed in the form of human capital. Human capital, which embodies the present value of future contribution payments, is accounted for because it resembles a series of bond-like cash flows. As a consequence, it is of relevance in rebalancing the risk exposure on the accumulated retirement savings. Since the desired benefit is aimed to be realized subject to a constraint of securing the guaranteed benefit, the strategy requires the sum of accumulated savings and discounted future contribution payments to be larger than or equal to the present value of the guarantee. Any integrated portfolio value remaining above the minimum value required to deliver the guarantee is available to be invested in equity to enhance the probability of arriving at the desired income target. If in the event of fortunate investment returns the sum of accumulated savings and discounted future contribution payments approximates the present value of the desired benefit, a lock-in strategy is performed by increasing the share of portfolio value invested in bonds at the expense of the share in equities.

The strategy to construct the payoff of the pension product allows implementation as a private investment scheme. However, a collective organization towards retirement security may be required. As the strategy to realize the target benefit entails an initial investment in risky assets by borrowing at the risk-free rate, a private implementation may be challenged by borrowing constraints due to young individuals' lack of collateral. Within a collective scheme, participants can ex-ante decide on the scheme's aggregated asset allocation and accordingly specify the ex-post division of the realized return on investment. As such, a collective organization serves to overcome restricted access of young individuals to capital markets. In addition, scale advantages present an important argument for organizing retirement security collectively. As the aggregated pension assets are managed with a single asset allocation, the scheme benefits from internal transactions and reduced transaction costs compared to individual trades. In addition, scale economies tend to reduce operational costs (Bikker and De Dreu, 2007). To illustrate the scheme's investment strategy, the aggregated asset allocation is displayed in the event a default pension product is replicated for all participants in the scheme.

Finally, the collar approach is compared to investment strategies more commonly executed in the pension industry. As the pension product is financed on a self-financing basis, the proposed strategy is financially fair. The property of financial fairness ensures that it is justified to compare the performance of the collar approach to alternative life cycle strategies. The investment strategies discussed all rebalance the asset allocation according to predefined schemes, which either provide decreasing or constant equity exposure over the life cycle. The results indicate that the collar approach outperforms the evaluated life cycle strategies in terms of the desired replacement rate that can be offered given an adverse scenario replacement rate. Moreover, the collar approach offers a larger probability of realizing the desired benefit given a predefined level of downside risk.

The paper is organized as follows. Section 2 specifies the financial market and labor market settings. The pension scheme settings are discussed in Section 3 and the pension product is illustrated in Section 4. The dynamic investment strategy to achieve the target benefit is presented in Section 5 and Section 6 compares the collar approach to alternative life cycle strategies. Section 7 concludes.

2 Market settings

2.1 Financial market

The investment opportunities are represented by the instantaneous riskless bond and the risky stock of a standard Black-Scholes market (Black and Scholes, 1973). The only risk factor is stock market risk, and it is traded through a stock market index S_t driven by Brownian motion. The riskless cash bond B_t is subject to deterministic exponential growth at a constant rate r. Specifically, the asset price processes are given by

$$\begin{aligned} dS_t &= \mu S_t dt + \sigma S_t dZ_t \\ dB_t &= r B_t dt \end{aligned} \tag{1}$$

where μ and σ are the constant drift and volatility parameters, and Z_t is a standard Brownian Motion. The assumed Black-Scholes market presents a highly simplified economy. In the analysis, we do not include several features that are often considered in life cycle investment literature, such as stochastic interest rates, stochastic inflation and asset return predictability. In a more comprehensive investigation, these factors should be taken into account; here the aim is to present a first analysis.

In subsequent sections, results are illustrated by numerical examples. For this purpose, it is assumed that the economy is characterized by a real riskless interest rate of 2%, expected real stock market return of 5% and stock market volatility equal to 18%. The financial parameter values are summarized in Table 1.

r	μ	σ
0.02	0.05	0.18

Table 1: Financial parameter values.

2.2 Labor market

The economy considered is populated by sixty generations which range from the age of 25 to 85. The population is assumed to be a homogeneous group of individuals who start working at age 25, retire at age 65, and have an expected remaining life time of twenty years. We assume all idiosyncratic mortality risk to be fully diversified and the expected improvement in life expectancy is included in the twenty-year retirement period. Moreover, it is assumed individuals participate in the pension scheme during both the active period and the retirement period. There are no individuals who enter late or leave prematurely. Let the parameters T_R and T_D denote the length of the active period and the full period individuals participate in the pension scheme, respectively.

During the active period individuals work and earn a flat real labor income Y. Labor income is assumed riskless and wage inflation is identical to price inflation. Moreover, labor market risks are absent. Part of the labor income is saved in the pension scheme for consumption after retirement. In retirement, individuals receive no income other than their pension benefit. The labor market parameters are summarized in Table 2.

T_R	T_D
40	60

Table 2: Labor market parameter values.

3 Pension scheme settings

For the purpose of illustration, a default pension product is replicated for all participants in the scheme. In this section, we therefore frame the pension scheme in terms of generational accounts, rather than individualized accounts. The default product considered is introduced in Section 4.4.

3.1 Contribution rate

In a complete market, such as the Black-Scholes market, there is a unique price for any contingent claim, namely the initial wealth needed to finance the replicating portfolio. The contribution rate of the pension scheme depends on the amount of initial wealth needed to replicate the payoff of the collar option. Since participants pay periodic contributions, rather than one single contribution, the sum of the discounted pension contributions is required to be equal to the price of the option in order to finance the option replication strategy. Given the assumption that participants contribute a constant fraction of labor income over the active period of the life cycle, the contribution rate c is solved from

$$V_0 = cY \int_0^{T_R} e^{-rs} ds \tag{2}$$

where V_0 denotes the unique price of the option upon entrance into the scheme. As a participant's wage Y is assumed to be constant in real terms, the discount rate equals the real riskless return r.

3.2 Accrual rate

The pension scheme is framed in terms of accrued pension income. Rather than monitoring participants' accumulated savings, the pension scheme keeps account of the amount of pension income participants receive in retirement. In fact, the amount of pension income is expressed in terms of a participant's real wage Y and resembles a deferred real annuity.

The accrual of pension income depends on the contribution rate in a financially fair way. Given a constant contribution rate, the financially fair accrual of pension income decreases over the life cycle as a consequence of the time value of money. Depending on the contribution rate c, the newly accrued pension income ΔP^x of an x-year-old participant is defined by

$$\Delta P^x = \varepsilon^x Y \tag{3}$$

where the pension accrual rate ε^x depends on the price of a deferred real annuity and is to be solved from

$$c = \varepsilon^x \int_{T_R}^{T_D} e^{-r(s-x)} ds. \tag{4}$$

3.3 Indexation policy

The indexation policy determines the dynamics of participants' accrued pension income. Since the targeted benefit is constructed by executing a replicating investment strategy, the indexation policy is return-driven. Moreover, indexation is cohort-specific as the replicating portfolio differs between generations. Depending on the percentage of a cohort's pension assets invested in risky assets α_t^x, indexation is granted based on realized returns. Let P_t^x denote the accrued pension income of age cohort x at time t, then the dynamics of the accrued pension income are given by

$$dP_t^x = \Delta P^x \cdot 1_{\{x<T_R\}} dt + P_t^x di_t^x \tag{5}$$

where di_t^x represents the dynamics of the indexation policy of age cohort x, and the indicator function $1_{\{x<T_R\}}$ ensures participants accrue new pension income only during the active period. The indexation process di_t^x is adapted to the Brownian motion Z_t and is defined by

$$di_t^x = \alpha_t^x(\mu - r)dt + \alpha_t^x \sigma dZ_t. \tag{6}$$

Since the growth rate of the pension liabilities is equal to the riskless interest rate r, the indexation factor depends on the realized excess return on stocks. As a result of the return-driven indexation policy, the accumulation of pension income exactly matches the growth of the underlying assets.

3.4 Liabilities

The liabilities of the pension scheme are equal to the market value of the participants' accumulated pension income. At time t, the liabilities corresponding to the accrued pension income of age cohort x are equal to

$$L_t^x = P_t^x \int_{\max\{T_R,x\}}^{T_D} e^{-r(s-x)} ds. \tag{7}$$

The total liabilities of the pension scheme are equal to the aggregated value of the cohort-specific liabilities. Total pension liabilities are given by

$$L_t = \int_0^{T_R} L_t^x dx. \tag{8}$$

3.5 Assets

The aggregated pension assets are equal to the contributions made into the scheme and the returns realized on investment. During retirement, pension assets are liquidated to pay out participants' pension benefits. Let A_t denote the value of the aggregated pension assets and let α_t denote the percentage of aggregated pension assets invested in risky assets, then the dynamics of the assets are given by

$$dA_t = \left[A_t(r + \alpha_t(\mu - r)) + C - P_t\right] dt + \alpha_t \sigma A_t dZ_t \tag{9}$$

where C denotes the amount of total contributions received and P_t denotes the value of the pension benefits paid at time t. More specifically, the definitions are

$$\begin{aligned} C &= cY \cdot T_R \\ P_t &= \int_{T_R}^{T_D} P_t^x dx. \end{aligned} \tag{10}$$

The amount of total contributions C is independent of time, because the contribution rate is constant and generations of pension fund participants are assumed to be homogeneous.

3.6 Asset allocation

The pension scheme manages the assets with a single asset allocation in order to achieve the benefit profile for the different age cohorts in the scheme. Given the replication strategies of the age cohorts, the asset allocation of the pension fund is determined such that all investment risk is absorbed by the participating cohorts. The investment risk is completely attributed to the cohorts if the asset allocation of the pension scheme equals the weighted average of the cohort-specific investment strategies according to the cohorts' stake in the liabilities. Therefore, the investment strategy of the pension fund is determined by

$$\alpha_t = \int_0^{T_D} \alpha_t^x \frac{L_t^x}{L_t} dx. \tag{11}$$

Given the indexation policy introduced in Section 3.3, the ex-post realized return on the assets is assigned to the age cohorts according to each cohort's replication strategy. Note that the aggregated asset allocation also depends on the investment strategy of the retired cohorts. For simplicity, it is assumed that the retirees' assets are fully invested in the riskless asset to finance the annuity payments.

3.7 Funding ratio

The funding ratio is defined as the ratio of the market value of the assets over the market value of the liabilities, i.e.

$$FR_t = \frac{A_t}{L_t}. \tag{12}$$

The funding ratio is commonly used to monitor the solvency position of pension funds. In the pension scheme employed in this paper, the market value of the liabilities always exactly matches the value of the pension assets. Hence, the funding ratio equals one at every point in time.

4 Product specification

4.1 Option strategy

A standard collar on a stock market index is established by purchasing the index, holding a put option at strike K_1, and writing a call option at strike $K_2 \geq K_1$. Moreover, the put and the call require the same expiration date. The strategy offers put protection in return for limited upside potential on the underlying asset as a result of writing the call. The premium received from writing the call partly offsets the costs of the put. Hence, by accepting limited upside, the investor is able to obtain downside protection at smaller cost than the cost of the put alone.

The pension product proposed in this paper is specified more generally than a standard collar strategy. Apart from setting the strike prices, the product allows choosing the floor and the cap. Let V_T be the value of the product at maturity T, and let θ_1 and θ_2 be the floor and the cap, respectively. Given the strike prices K_1 and K_2, the payoff of the pension product is defined by

$$V_T = \max\left\{\theta_1, \min\left\{\theta_1 + \frac{\theta_2 - \theta_1}{K_2 - K_1}(S_T - K_1), \theta_2\right\}\right\}. \tag{13}$$

Similar to the standard collar, the product's payoff function may be established by a linear combination of the stock market index, a protective put option and a covered call option.[2] As an alternative, however, we use the put-call parity to present the option composition by the value of the floor and the difference of two call options. As a result, the payoff function takes the straightforward representation given by

$$V_T = \theta_1 + a\left(F(S_T, K_1) - F(S_T, K_2)\right) \tag{14}$$

where $a = \frac{\theta_2 - \theta_1}{K_2 - K_1}$ represents the slope of the linearly increasing part of the option payoff and $F(S_T, K) = \max\{S_T - K, 0\}$ denotes the payoff function of a call option. Figure 1 illustrates the product's payoff function and the option decomposition.

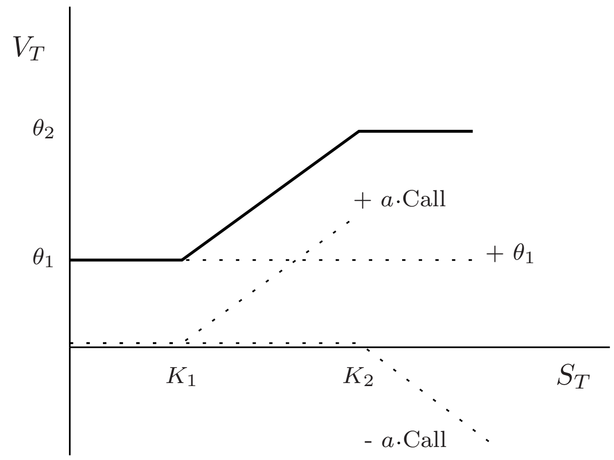


Figure 1: Option composition of the pension product.

4.2 Option parameters

The pension product aims to provide participants with real lifetime income in retirement. Therefore, the floor and the cap of the option are defined by the price of real annuities. Whereas the floor corresponds to a minimum, or guaranteed replacement rate of final pay, the cap corresponds to a maximum, or desired replacement rate. Let the *guarantee* (κ_1) be defined as the minimum replacement rate and let the *ambition* (κ_2) be defined as the maximum replacement rate. Then, the floor and the cap of the option are given by

$$\theta_i = \kappa_i Y \int_0^{T_D - T_R} e^{-rs} ds, \qquad i \in \{1, 2\}. \tag{15}$$

The strike prices K_1 and K_2 determine for which values of the stock market index at maturity, the option pays the guaranteed or the desired level of pension income. To give meaningful interpretation to the strike prices, the parameter values are expressed in terms of the probabilities of reaching the ambition level and ending up with the guaranteed level. Hence, the strike prices reflect the likelihood

[2] $V_T = (aS_T + b) + a(\max\{K_1 - S_T, 0\} - \max\{S_T - K_2, 0\})$, where $a = \frac{\theta_2 - \theta_1}{K_2 - K_1}$ and $b = \theta_1 - aK_1$.

of realizing the ambition and the risk of ending up with the guarantee. The lower and upper strike are solved from

$$\begin{aligned} p_1 = \mathbb{P}(S_T \le K_1) &= 1 - \Phi\left(\frac{\log\frac{S_0}{K_1} + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \\ p_2 = \mathbb{P}(S_T \ge K_2) &= \Phi\left(\frac{\log\frac{S_0}{K_2} + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \end{aligned} \tag{16}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, and p_1 and p_2 represent the probabilities of ending up at the guarantee and realizing the ambition, respectively. Note that the probability of ending up with the guarantee is increasing in K_1, whereas the probability of realizing the ambition is decreasing in K_2.

4.3 Option price

The contribution rate is determined by the value of the option, which depends on the option parameters. In this section, an explicit formula for the value of the collar option before maturity T is presented. In the complete Black-Scholes market, risk-neutral pricing techniques can be used to determine the unique price for a contingent claim. Consequently, the risk-neutral pricing formula is used to price the collar option.

Let $\mathbb{Q}$ be the unique equivalent martingale measure corresponding to the bond as numéraire. According to the fundamental theorem of asset pricing, the process defined by $\{\frac{V_t}{B_t}\}$ is a martingale under the equivalent measure $\mathbb{Q}$ and therefore the value of the option at time $t \le T$ satisfies

$$V_t = E_t^{\mathbb{Q}}\left[\frac{B_T}{B_t}V_T\right] = e^{-r(T-t)}E_t^{\mathbb{Q}}V_T. \tag{17}$$

Solving for the option value at time t yields a function depending on time t and of the value on the underlying stock market index S_t at time t. The value of the collar option at time t is given by

$$V_t = aS_t(\Phi(d_1) - \Phi(d_2)) + e^{-r(T-t)}\Big(\theta_1 - a(K_1\Phi(d_1 - \sigma\sqrt{T-t}) - K_2\Phi(d_2 - \sigma\sqrt{T-t})\Big) \tag{18}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function,

$$d_i = \frac{\log(\frac{S_t}{K_i}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \qquad i \in \{1, 2\} \tag{19}$$

and $a = \frac{\theta_2 - \theta_1}{K_2 - K_1}$. Upon entrance into the pension scheme, the value of the collar option is equal to V_0. Section 3.1 describes the dependence of the contribution rate on the initial option price.

4.4 Parameter trade-offs

Given the contribution rate participants can afford to save, the product's parameters may be chosen. To present insight in the set of feasible choices, the trade-off between the ambition, the guarantee and the likelihoods of realizing the ambition and ending up with the guarantee is illustrated for a range of contribution rates. The relationship between the option parameters and the contribution rate is illustrated by numerical examples. The market parameters used are mentioned in Tables 1 and 2.

The default product considered is defined by the parameter values presented in Table 3. The guarantee and the ambition are equal to 50% and 80% of a participant's real wage, and the probabilities of ending up at the guarantee and realizing the ambition are equal to 2.5% and 70%, respectively. The contribution rate corresponding to the option parameters equals 17.5% of wage Y.

κ_1	κ_2	p_1	p_2	c
0.5	0.8	0.025	0.7	0.175

Table 3: Default option parameter values.

Subsequent subsections illustrate the trade-offs for a range of contribution rates. The focus is on two types of trade-offs:

i *Given fixed likelihoods, which ambition and guarantee may be chosen?*
Assume participants require a pension product that realizes the ambition with 70% probability and incorporates a risk of ending up with the guarantee equal to 2.5% probability. Given a contribution rate, participants face a trade-off between the ambition and the guaranteed that can be offered.

ii *Given fixed ambition and guarantee, which likelihoods may be chosen?*
Assume participants require a pension product whose ambition level is equal to 80% and whose guaranteed level is equal to 50%. Given a contribution rate, participants face a trade-off between the probability of realizing the ambition and the probability of ending up with the guarantee.

Note that alternative parameter trade-offs are possible. For these trade-offs, we refer to appendix A.

4.4.1 Trade-off in terms of ambition and guarantee

The strike prices are fixed such that the ambition level and guaranteed level are realized with 70% and 2.5% probability, respectively. Figure 2 illustrates the trade-off between the ambition level and the guaranteed level for a given contribution rate in terms of the option payoff scheme. In addition,

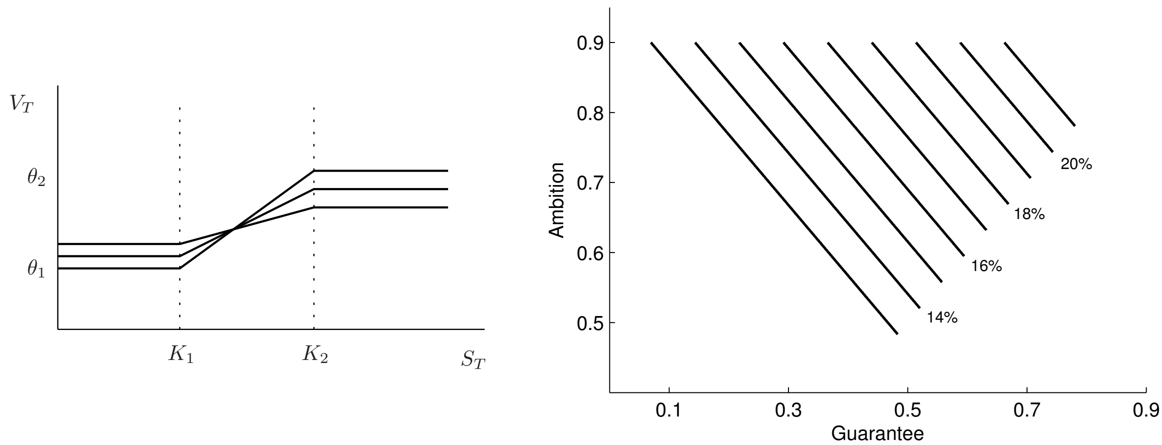
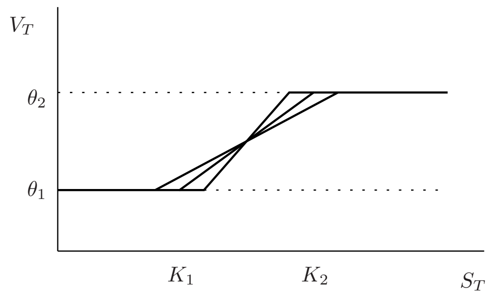


Figure 2: Trade-off in terms of ambition and guarantee.

the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves. The curves show that as the floor increases, more insurance is needed in the bad-state region, and accordingly the cap in the good-state region decreases to satisfy the budget constraint imposed by the constant contribution level. Given the likelihoods assumed in the default, the slope of the curves shows that increasing the ambition is approximately equally expensive in terms of giving up guarantees as the other way around. Hence, for the default option the guarantee increases from 50% to 60% in case the ambition decreases from 80% to 70%. Note that the slope of the trade-off curves depends on the chosen strike prices. Ceteris paribus, the slope is decreasing in K_2 and increasing in K_1. For example, if the probability of realizing the desired pension benefit increases, the upper strike K_2 decreases and therefore the slope of the trade-off curves increases. Accordingly, increasing the ambition becomes more expensive in terms of giving up guarantees. Furthermore, figure 2 displays the additional costs of increasing the guarantee or the ambition. Since the trade-off curves corresponding to the default product have equal slope and are approximately equidistant, the trade-off can be summarized by stating that for 1 percentage point additional contribution, the guarantee, the ambition, or a combination of both may be increased by 7.4 percentage points in total.

4.4.2 Trade-off in terms of likelihoods

The guarantee and the ambition are fixed at replacement rates equal to 50% and 80%, respectively. Figure 3 illustrates the trade-off between the strike prices for a given contribution rate in terms of the option payoff scheme. In addition, the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves. The curves show that as the upper strike price K_2 decreases, more states pay the ambition level and therefore the probability of realizing the ambition increases. As a consequence, the amount of bad states paying the guaranteed level has to increase in order to maintain a constant contribution level. This implies that the lower strike price K_1 increases and thus the probability of ending up with the guarantee increases. Given the ambition and guarantee assumed in the default, the trade-off curves show that increasing the probability of realizing the ambition is slightly more expensive in terms of the probability of ending up at the guarantee. For the default



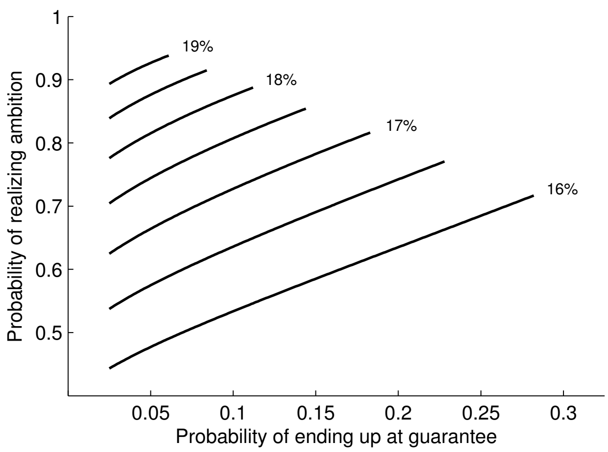


Figure 3: Trade-off in terms of likelihoods.

option, the probability of realizing the ambition increases from 70% to 80%, in case the probability of ending up at the guarantee increases from 2.5% to 10%. Furthermore, the figure shows that the upward shift of the trade-off curve decreases as the contribution increases. This implies that the additional cost of increasing the probability of realizing the ambition rises as the contribution rate increases. A similar conclusion holds for decreasing the probability of ending up at the guarantee. Hence, the marginal improvement of the probabilities, as a result of additional contribution, decreases.

5 Dynamic portfolio selection

In a complete market, such as the Black-Scholes market, every contingent claim can be replicated perfectly by suitable trading in the underlying asset. The initial wealth required to finance the replicating portfolio is equal to the unique price of the claim. Upon entrance into the scheme, however, the initial wealth is not available as participants pay periodic contributions rather a single one. Therefore, the risk exposure on the accumulated retirement savings has to be adjusted to perform the replication strategy. In this section, we discuss the replicating asset allocation and the risk exposure resulting on the accumulated retirement savings. Finally, the aggregated asset allocation of the fund is illustrated.

5.1 Replicating asset allocation

The replicating portfolio is rebalanced by executing a dynamic investment strategy. The strategy involves maintaining a position in the underlying asset such that the replicating portfolio mimics the option value over time. To achieve the exact replication, at any instant the delta of the replicating portfolio has to be equal to the delta of the option. Mathematically, the delta δ_t^x is the first derivative of the option value with respect to the underlying asset value, i.e.

$$\delta_t^x = \frac{\partial V_t^x}{\partial S_t}. \tag{20}$$

The option delta's value represents the volume of stocks to be invested in. Any portfolio value remaining, is required to be invested in the riskless bond. The pension product's delta is equal to the difference of the deltas corresponding to the call options the product may be decomposed in, i.e.

$$\delta_t^x = a\left(\Phi(d_1^x) - \Phi(d_2^x)\right). \tag{21}$$

Since the replicating portfolio value is equal to the present value of the option V_t^x, the fraction of portfolio value invested in stocks is required to be equal to

$$\phi_t^x = \frac{\delta_t^x S_t}{V_t^x}. \tag{22}$$

Figure 4 illustrates the replicating asset allocation corresponding to the default option by representing the delta as a function of time and annualized stock return.[3] The volume of stocks invested in, reduces to zero in case the value of the replicating portfolio approximates the discounted value of either the cap or the floor. As the time to maturity decreases, the option delta displays an increasingly discrete behavior; the delta either tends to zero or to a, the value of which represents the slope of the linearly increasing part of the option's payoff function. Furthermore, figure 4 displays the fraction of

[3] The value of the delta presented in Figure 4 corresponds to wage $Y = 1$ and initial stock price $S_0 = 1$.

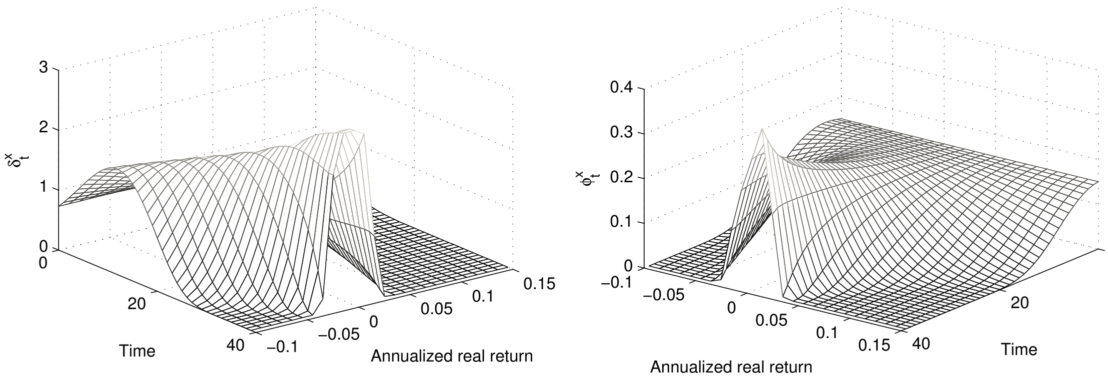


Figure 4: Option delta and stock exposure of replicating portfolio.

replicating portfolio value invested in stocks, i.e. ϕ_t, as a function of time and annualized stock return. For the default product assumed, the figure shows that the stock weight at the beginning of the investment period is approximately equal to 15% initial wealth. Thereafter, the stock weight is continuously adjusted depending on realized returns and time to maturity. The dynamics are explained similarly to the relation between the option delta and the realized return. In case stock returns turn out to be either very fortunate or adverse, the fraction of portfolio value invested in stocks tends to reduce to zero. If, on the other hand, the payoff is likely to realize between the product's cap and floor, then the stock weight of the replicating portfolio tends to increase and may approximate 40%.

5.2 Investment strategy

Upon entrance into the scheme, participants have no accumulated retirement savings yet. In contrast, participants own the required initial wealth fully in terms of discounted future contribution payments. As the time to retirement decreases, participants' human capital is gradually transformed into financial capital to save for income in retirement. To arrive at the income target, the strategy requires the sum of the accumulated savings and discounted future contributions to be equal to option's present value.

Human capital is non-tradable. Therefore, the risk exposure on the accumulated savings has to be adjusted to achieve the replicating asset allocation on the integrated portfolio of accumulated savings and future contributions. Since the accumulated retirement savings of age cohort x are equal to the value of their pension entitlements L_t^x, the fraction of savings invested in stocks has to be equal to

$$\alpha_t^x = \frac{\delta_t^x S_t}{L_t^x}. \tag{23}$$

Figure 5 illustrates the investment strategy corresponding to the default option by representing the stock weight as a function of time and annualized real return. The left plot shows that participants are initially short in the risk-free rate to finance the equity investments required to arrive at the desired income target with the predefined likelihood. Young participants can afford to have leveraged stock exposure because the discounted value of their future contribution payments exceeds the present value of the product's floor. In other words, their stock of human capital is large enough to construct the guaranteed income level by using anticipated future contributions only. The fraction of accumulated

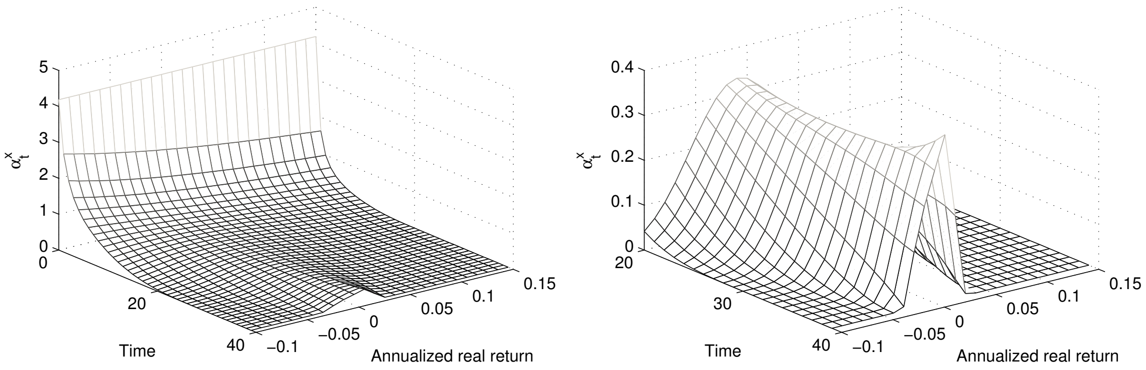


Figure 5: Stock exposure on accumulated savings.

savings invested in stocks does not drop below 100% before either human capital becomes insufficient, or the value of the integrated portfolio of accumulated savings and future contributions equals the discounted value of the product's ambition level.

Based on the parameters of the default pension product, it holds that $\delta_t^0 S_t = 0.72 \cdot Y$. This implies that upon entrance into the pension scheme, participants borrows 72% of their annual wage to invest in the stock market. As time increases, the fraction invested in stocks decreases rapidly. After five years, the stock weight is likely to be less than 100%. The right plot displays the stock weight for the last twenty years only. Depending on the realized returns on investment, the fraction invested in stocks varies between 0% and approximately 40% during the final years of the active period.

Similarly to the life cycle models discussed by Bodie, Merton and Samuelson (1992) and Teulings and De Vries (2006), equity investments are initially financed by borrowing against the risk-free rate. However, these utility-based life cycle models find a much larger amount participants may want to borrow to acquire the optimal asset allocation. The studies indicate that young participants may want to borrow as much as six times their annual wage to invest in the equity market.

In practice, borrowing against the riskless assets may be hard to realize since young participants face credit constraints due to a lack of collateral. On the other hand, a rationale for collective pension schemes is that borrowing constraints for young participants may be alleviated (Bovenberg, Koijen, Nijman and Teulings, 2007). Within a collective pension scheme, participating cohorts may ex-ante decide on the fund's aggregated asset allocation and accordingly specify the ex-post division of the realized return on investment. As such, a collective organization serves to overcome restricted access of young individuals to capital markets.

5.3 Aggregated asset allocation

The pension scheme manages the pension assets with a single asset allocation to execute the replication strategies of the cohorts in the scheme. Figure 6 illustrates the asset allocation of the fund together with the corresponding stock value for three scenarios over a 40-year horizon. The dependence of the asset allocation of the fund on the stock value is explained similarly to the relation between the stock and the replication strategy. In case the fund experiences either favorable or adverse returns, the aggregated asset allocation in stocks is reduced in favor of bonds. Furthermore, figure 6 presents the expected asset allocation and the 10% and 90% quantiles, which are given by the dotted lines. The figure shows that the fraction of pensions assets the fund is expected to invest in stocks is approximately equal to 12%. The quantiles indicate that the fraction may vary between 4% and 20%.

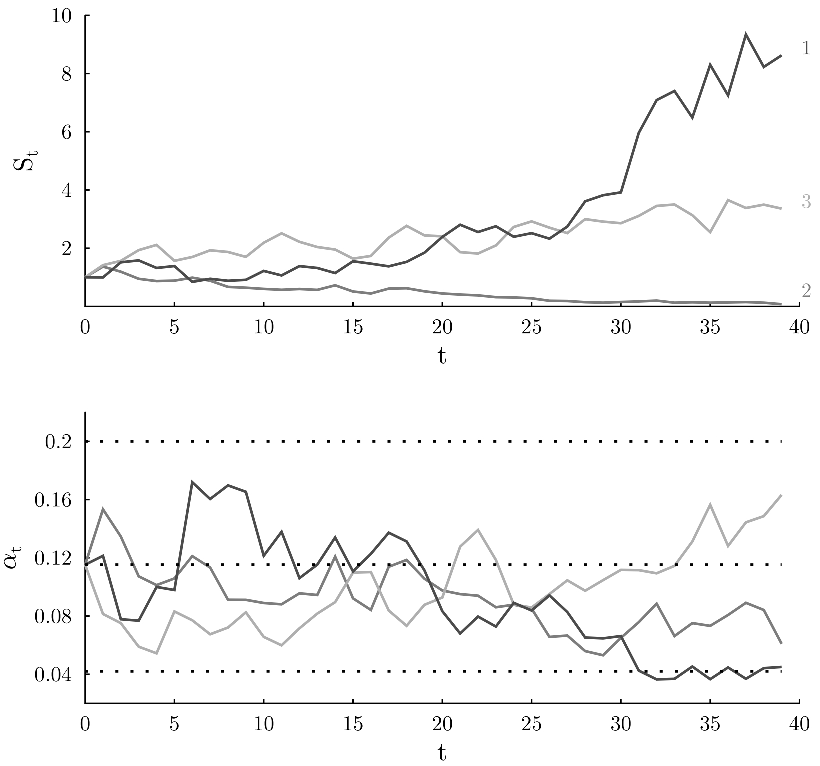


Figure 6: Asset allocation of the pension scheme.

On aggregated fund level, the asset allocation corresponding to the replication strategy, may be characterized as conservative compared to the current asset allocations of Dutch pension funds. Before the latest financial crisis, Dutch pension funds invested approximately 50% of their assets in stocks (Broeders and Rijsbergen, 2010). In fact, the asset allocation of the scheme analyzed in this paper, shows a stronger resemblance to the investment strategies more commonly implemented twenty years ago, when Dutch funds invested 20% of the pension assets in equities. In view of the characteristics of the pension product offered, it does not seem very remarkable that the asset allocation towards stocks is low. The proposed pension product guarantees a minimum payoff which requires a significant investment in fixed-income securities to make sure at least the guaranteed income level is realized. Moreover, the assets of retirees in the fund are fully financed by investments in the riskless cash bond.

On the other hand, the fraction invested in stocks may increase in case asset return predictability and stochastic interest rates are accounted for. To address this question, a more comprehensive financial model has to be analyzed in future research.

6 Retirement income

6.1 Return-driven indexation

The indexation policy of the pension scheme is return-driven and cohort-specific. Depending on the percentage of a cohort's pension assets invested in risky assets α_t^x, indexation is granted based on realized returns. Since the growth rate of the pension liabilities is equal to the riskless interest rate r, the indexation factor depends on the realized excess return on stocks. As a result of return-driven indexation, the accumulation of pension income exactly matches the growth of the underlying assets.

Figure 7 presents the indexation policy over the active period of the life cycle. The drawn line represents the expected factor of indexation, and the dotted lines provide the 1% and 99% quantiles. In addition, the annual indexation is displayed for a randomly selected scenario. If indexation is equal to one, this implies no excess return on investment is realized and therefore the entitlements of participants remain constant in real terms. The quantiles indicate that indexation can deviate from the a priori expected factor of indexation. Particularly early in life, indexation may be volatile due to equity exposure in excess of 100%. While the upper quantile shows that the pension entitlements of cohorts may double or even triple, the lower quantile indicates that indexation might be negative. As time increases, the indexation policy becomes less volatile because stock exposure is reduced.

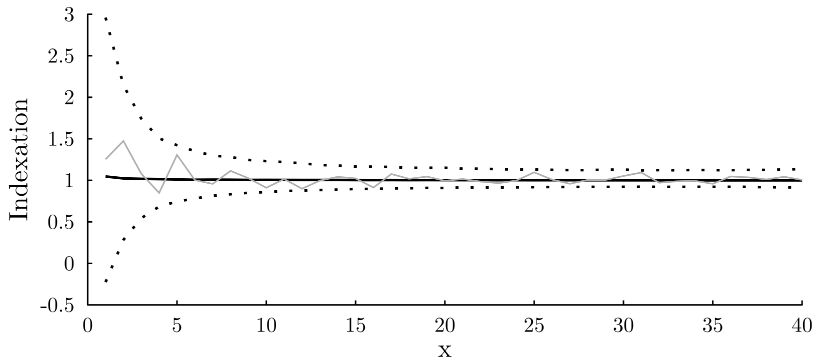


Figure 7: Return-driven indexation.

As a consequence of negative indexation, participants accumulate debt rather than wealth. Young participants may therefore face negative pension entitlements. The possibility of negative pension entitlements presents a potential problem for the pension product. In current labor markets, employees are becoming increasingly mobile as they more often shift between employers. In case a young employee who experienced adverse equity returns decides to switch jobs, he faces the obligation to pay off the debt corresponding to the pension product offered by the pension fund he used participate in.

One way the problem may be resolved, is to enforce mandatory retirement saving. A system in which participation is obligatory helps to secure human capital of young individuals as collateral (Bovenberg et al., 2007). In the event of a career switch, negative pension entitlements are transferred to the new employer's fund and subsequently debt can be paid off by using part of the stock of human capital remaining. Alternative solutions have to be determined for individuals mandatory participation does not apply to, such as self-employed. For instance, research could be aimed at the creation of a target benefit given short-selling constraints. Finally, it is addressed that leveraged equity exposure may be reduced in case human capital embodies stock-like properties. As such, the value of future contributions is risky and therefore less stock exposure on accumulated savings is expected to be required initially. Studies of Cocco, Gomes and Maenhout (2005) and Benzoni, Collin-Dufresne and Goldstein (2007) discuss the concept of future wage income with stock-like properties.

6.2 Accumulated income

Participants accrue pension income from contribution payments and return-driven indexation. Figure 8 illustrates the accrued income as a function of time and annualized real stock return. For ease of viewing, the accrual is displayed over the last twenty years only. The bold line depicts the payoff scheme of the option in terms of the benefit at retirement. It is shown that the surface tends to display an increasingly discrete dependence on the realized return as the time to retirement decreases. This directly results from the development of the option value over time. As the time to maturity decreases, the value function increasingly tends to the piecewise linear payoff function. Furthermore, Figure 8 displays the probability distribution of the pension benefit at retirement. In addition to the predefined probabilities of realizing the ambition and ending up at the guarantee, the figure shows that the probability mass between the ambition and the guarantee turns out to be approximately equally divided.

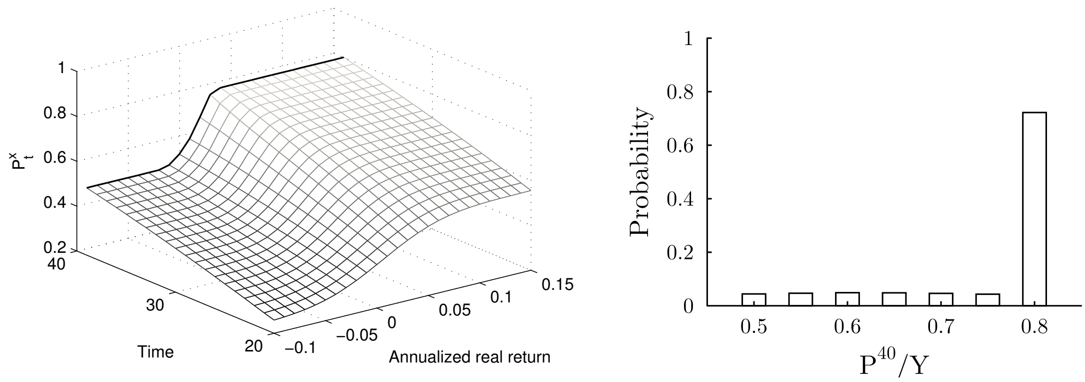


Figure 8: Distribution of pension income.

6.3 Retirement planning

The strategy is designed to achieve the target benefit agreed on. However, as time passes and asset prices move, the probability of realizing the desired income and the probability of ending up with the guarantee may deviate from the predefined probabilities. Figure 9 displays the dynamics of the probabilities as a function of time and annualized stock return. To inform participants about the progress of their income accrual for retirement, the likelihoods of arriving at the desired income level and of ending up with the guarantee may be communicated periodically during the accumulation phase. For example, after 10 years of annualized real return equal to zero, participants have seen the probability of realizing the ambition decrease from 70% to 60% and the probability of ending up with the guarantee increase from 2.5% to 2.75%. Based on updated information, participants can decide whether or not their planning for retirement is on track. In the event the likelihood of realizing the desired income is not regarded to be sufficient, participants can take measures to improve the changes of achieving their income goal. To enhance this probability, participants may decide to save additionally or to work longer.

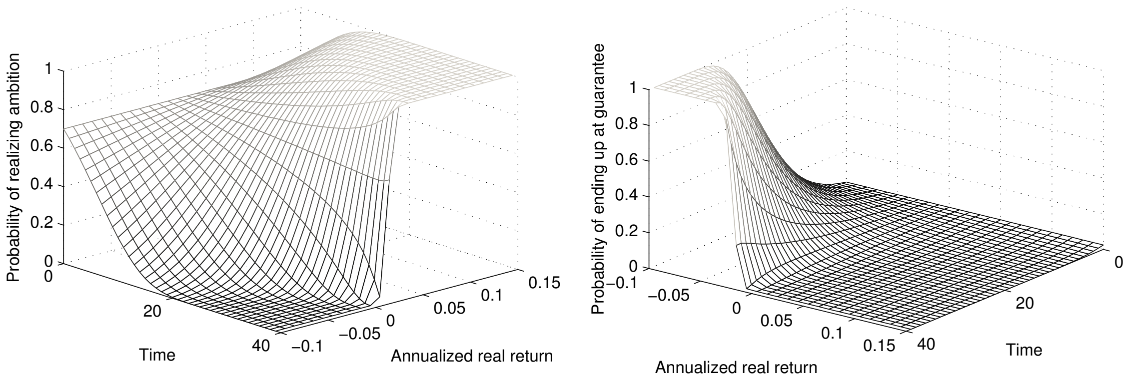


Figure 9: Probability of realizing the ambition and ending up at guarantee.

7 Comparison to life cycle strategies

In this section, the performance of the collar approach is compared to life cycle strategies more commonly executed in the pension industry. The life cycle strategies discussed are classified as target date schemes or myopic schemes. All strategies rebalance the asset allocation according to predefined schemes which either provide decreasing or constant equity exposure over the life cycle. The focus is on two types of performance measurement:

i *Given fixed likelihoods, which desired and minimum required benefit can be offered?*
It is investigated which desired and minimum required pension benefit a strategy offers, given the prerequisites that the desired benefit is realized with 70% probability and the risk of falling short of the minimum required benefit does not exceed 2.5% probability.

ii *Given fixed desired and minimum required benefit, which likelihoods can be offered?*
The desired benefit and the minimum required pension benefit are assumed to be equal to 80% and 50% of a participant's wage. It is investigated how much probability a strategy assigns to realizing the desired benefit and to falling short of the minimum required benefit.

The following sections discuss the life cycle strategies and the performance measures in detail.

7.1 Schemes

The target date schemes start out with 100% equity exposure and shift to the riskless portfolio at retirement. The speed with which the different schemes reduce the equity exposure may vary over the life cycle. Figure 10 depicts the age-related stock exposure α_t^x corresponding to the schemes evaluated. Three strategies are emphasized and referred to as *risk averse*, *linear* and *risk seeking*. These schemes reduce the equity exposure quickly early in life, on a straight-line basis, or not until later in life, respectively. In the next section, the performance of these schemes is illustrated explicitly.

Myopic schemes have constant equity exposure over the life cycle. The performance of the schemes given by 10%, 30%, 50% and 70% constant equity exposure are illustrated in Appendix B.

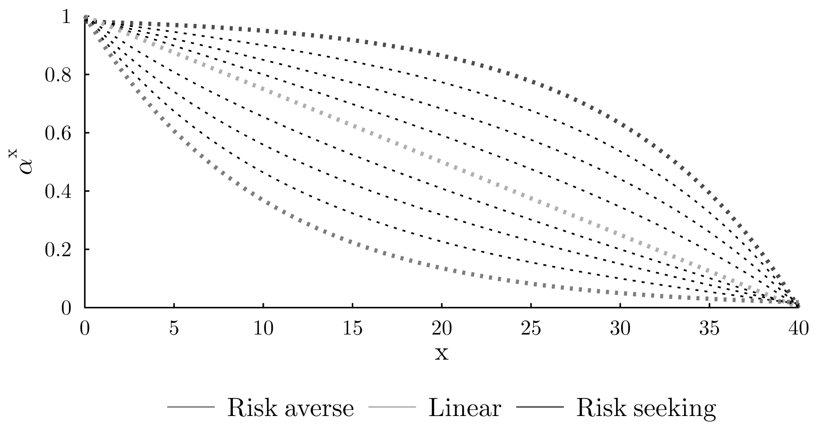


Figure 10: Target date schemes.

7.2 Performance measurement

In this section, the contribution rate is assumed to be equal to 17.5% of wage Y, similarly to the contribution rate corresponding to the default pension product introduced in chapter 4.

7.2.1 Performance in terms of desired and minimum required benefit

Assume plan participants require the desired level of pension income to be realized with at least 70% probability. Moreover, the probability of falling short of the minimum required income level is not allowed to exceed 2.5% probability. Given these prerequisites, it is investigated which desired and minimum required replacement rate the implementation of a life cycle strategy may offer.[4]

Every target date scheme provides a desired and minimum required replacement rate. Together, as a class of strategies, the set of target date schemes yields a trade-off curve. The curve is displayed in Figure 11. It is shown that as the target date schemes allocate a larger fraction of portfolio value towards stocks, the minimum required replacement rate decreases, while the desired replacement rate remains approximately constant. In addition, Figure 11 displays the trade-off curve for the collar approach given the requirement that the desired and guaranteed income are realized with 70% and 2.5% probability, respectively. The figure shows that given a minimum required replacement rate, the collar approach offers a larger desired replacement rate than the target date schemes offer. In addition, it is displayed that the collar approach may offer desired replacement rates that are unattainable in

[4] Formally, the desired replacement rate is defined by $\max\{\nu \geq 0 \mid \mathbf{P}(P^{40}/Y \geq \nu) \geq 70\%\}$ and the minimum required replacement rate by $\max\{\nu \geq 0 \mid \mathbf{P}(P^{40}/Y \leq \nu) \leq 2.5\%\}$.

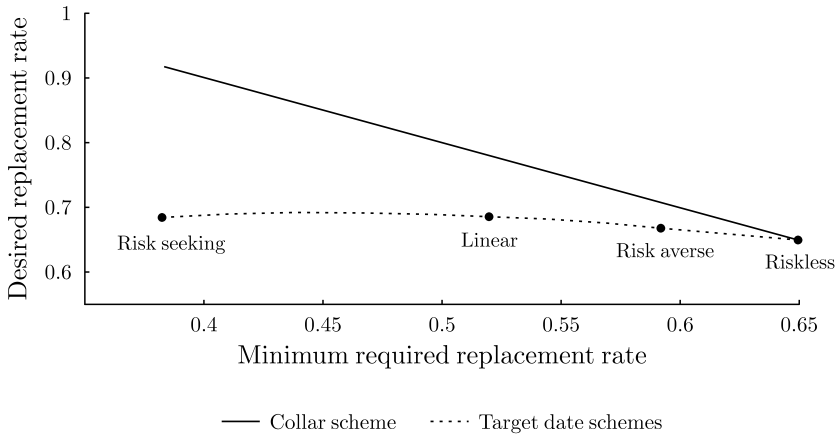


Figure 11: Trade-off between desired and minimum required replacement rate.

case a target date scheme is executed. Appendix B displays the trade-off curve for the myopic schemes. The curve shows a strong resemblance to the curve associated with the target date schemes.

Note that the collar approach provides a maximum and a minimum pension benefit. The income offered by life cycle strategies is not bounded by a floor or a cap. Participants face the risk of ending up with a benefit lower than a minimum required replacement rate, but also have the upside potential to realize pension benefits well in excess of the wage earned in the active period. However, if participants require a substantial level of certainty in realizing the desired benefit, then the collar approach outperforms the life cycle strategies in terms of the desired replacement rate that can be offered given a predefined adverse scenario replacement rate. Conversely, given a desired replacement rate, a larger minimum required replacement rate may be offered.

7.2.2 Performance in terms of likelihoods

Assume participants require a desired and minimum required income equal to 80% and 50% of wage Y, respectively. Given these prerequisites, it is investigated how much probability target date schemes assign to realizing the desired benefit and to falling short of the minimum required benefit. Similarly to previous section, a trade-off curve is constructed. The curve is displayed in Figure 12. It is shown that the risk averse scheme has zero probability of falling short of the minimum required benefit. However, as a consequence of conservative equity exposure the scheme offers only 10% probability of realizing the desired level. As the target date schemes allocate a larger fraction of portfolio value towards stocks, the probability of realizing the desired replacement rate increases at the expense of the probability of falling short of the minimum required replacement rate. In addition, Figure 12 displays the trade-off curve for the collar approach. The floor and the cap correspond to replacement rates equal to 50% and 80%, respectively. The figure shows that given a probability of shortfall (c.q. realizing the guarantee), the collar approach offers a larger probability of realizing the desired replacement rate than the target date schemes offer.[5] Appendix B displays the trade-off curve for myopic schemes. Again, the curve shows a strong resemblance to the curve associated with the target

[5]In fact, for the collar approach the axes are given by $\mathbf{P}(P^{40}/Y = 50\%)$ and $\mathbf{P}(P^{40}/Y = 80\%)$.

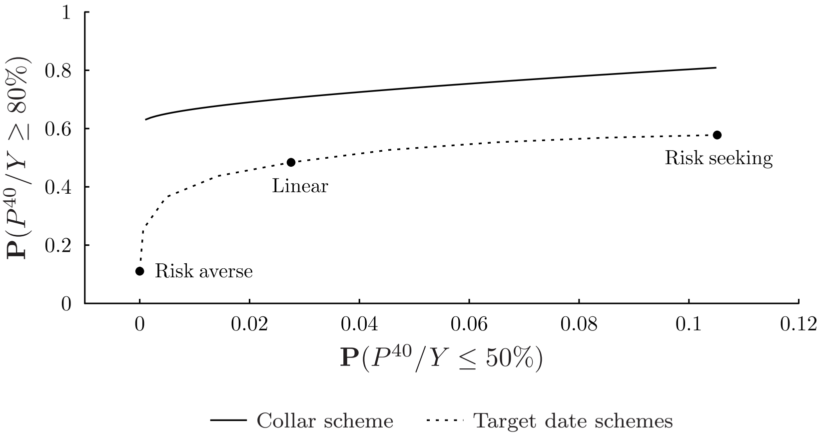


Figure 12: Probabilities of realizing desired and minimum required replacement rate.

date schemes. Concluding, given a probability of falling short of the minimum required benefit (c.q. realizing the guarantee), the collar approach outperforms the evaluated life cycle strategies in terms of the probability of realizing the desired benefit.

8 Conclusion

In this paper, a pension product is developed that is explicit about the pre-established goals that are aimed for. The proposed product presents a trade-off that is transparent in terms of required contributions, the income level targeted and guarantees offered. Depending on participants' preferences, individualized investment plans are constructed that focus on achieving the desired standard of living in retirement with a likelihood that is ex-ante defined. In addition, the event of falling short of the goal is managed. To address the risk of failure to achieve the desired income target, guaranteed income is offered to safeguard a minimum standard of living in retirement. Given the contribution rate participants can afford to save, the set of product parameters has to be chosen. Apart from choosing the desired and guaranteed level of income, participants are asked to determine the likelihood of realizing the desired benefit and the likelihood of ending up with the guarantee.

Karl Borch, who was one of the leading proponents of utility functions as a tool in actuarial science, has reflected on the use of expected utility (Borch, 1961) by contrasting two characters in a piece by the French playwright Jean Giraudoux. On the one hand, we have Cassandra, who can foretell the future; on the other hand, there is the beautiful Helen who can also see the future, but only the very happy and the very dreadful events. He likens the position of Helen to the view expressed by the noted economist George L.S. Shackle on the way in which businessmen arrive at their decisions: not by weighing all possible outcomes as in expected utility theory, but rather by focusing on the best outcome that can be reasonably hoped for as well as on to the worst outcome that is sufficiently likely so that it has to be taken into account (Shackle, 1947). The expected utility paradigm has been the dominating framework for academic work on portfolio selection in the past decades (Bodie et al., 1992). In this paper, we return in a sense to the beautiful Helen by representing the choices to be made in terms of two focal values, namely the desired level of benefits and the subsistence level that can be guaranteed. This might be called “framing” (Kahneman and Tversky, 1979), but there is an element of framing as well in restricting discussion to a parametric class of utility functions as is often done in the literature. By working with focal values, the decision that is to be made can be represented in an accessible way to those who have to make the decision, namely the ones who are contributing to the pension scheme and who expect benefits from it. The role of the financial engineer in this setting is to construct a parameterized set of policies that allows a good trade-off curve.

To assess the performance in realizing the targets, the approach developed in this paper is compared to investment strategies more commonly executed in the pension industry. As the pension product is financed on a self-financing basis, the proposed strategy is financially fair. The property of financial fairness ensures that it is justified to compare the performance of the collar approach to alternative life cycle strategies. The investment strategies discussed all rebalance the asset allocation according to predefined schemes, which either provide decreasing or constant equity exposure over the life cycle. The results show that the collar approach outperforms the evaluated life cycle strategies in terms of the desired replacement rate that can be offered given an adverse scenario replacement rate. Moreover, it is shown that given a predefined probability of falling short of the minimum required benefit (c.q. realizing the guarantee), the collar offers a larger probability of realizing the desired benefit.

Furthermore, it is illustrated that the aggregated asset allocation of the fund is characterized by a risk profile that is conservative compared to the current asset allocation of typical Dutch pension funds. Whereas Dutch funds invest approximately 50% of their assets in equities, the scheme analyzed in this paper is expected to invest only 12% of the assets in equities. In fact, the asset allocation of the scheme shows a stronger resemblance to the investment strategies more commonly implemented twenty years ago. The aggregated asset allocation indicates that the guarantee to realize a minimum payoff requires a significant investment in fixed-income securities. The conservative asset allocation may, however, present a disadvantage form a macroeconomic perspective. Low risk taking at the expense of risk-bearing capital may be unfavorable as it could harm innovation and economic growth (Boeri et al., 2006). Note that the fraction invested in equities may increase in case a more comprehensive model is analyzed that accounts for asset return predictability and stochastic interest rates.

To conclude, a few limitations of the scheme are addressed and some points are put forward for further research. First of all, the implementation of the collar approach may be challenged by the initial requirement to finance risky equity investments by borrowing against the risk-free rate. Although the collective organization of the pension scheme could serve to overcome restricted access of young individuals to capital markets, young cohorts are exposed to the risk of losing wealth in excess of their accumulated savings as a consequence of the leveraged equity exposure. The possibility of pension debt at young ages presents a potential problem for the approach even though a guaranteed benefit is ensured at retirement. In case a young employee who experienced adverse returns decides to become self-employed or switches jobs he faces the obligation to pay off the debt associated with the pension product offered by the fund he used to participate in. To resolve the problem, future research could be aimed at the creation of a target benefit given short-selling constraints.

Second, the collar approach is developed in a highly simplified representation of the real financial market. The Black-Scholes economy incorporates only stock market risk and assumes the presence of a cash bond subjected to constant interest accrual. In real markets however, long-dated pension claims are exposed to large interest rate risk as a result of time-varying interest rates. Moreover, inflation risk has to be addressed. As inflation may strongly erode the real value of pension savings in the long-run, inflation-indexed instruments have be include in the analysis to be able to construct a minimum income level in retirement. For further research, it is therefore recommended that the replication strategy is analyzed in a financial model that takes into account stochastic interest rates and stochastic inflation. As an additional source of risk, longevity risk should be considered.

Finally, a reflection is cast upon the delta replication strategy. As discussed, the investment strategy to replicate the payoff of a contingent claim requires a complete market setting and involves continuously rebalancing portfolio weights as time passes and the underlying assets move. At financial markets, however, transaction costs and non-traded risks may prevent portfolio managers from performing the perfect hedge. As a consequence, financial market insurance may fail to realize the predefined target benefit participants desire. For the future, research is recommended to be aimed at exploiting the potential of collective pension funds to complete markets. As participants are heterogeneous in terms of age and preferences regarding retirement goals, savings portfolios may have different optimal exposure to the risk factors at hand. As a consequence, internal markets could serve to trade risk exposure internally and mismatch risk in realizing a target benefit may be reduced.

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Appendices

A Alternative trade-offs

Cap and upper strike

Figure 13 illustrates the trade-off between the ambition level and the probability of realizing the ambition in terms of the option payoff scheme. In addition, the trade-off curves are depicted for a range of contribution rates, which are displayed next to the trade-off curves.

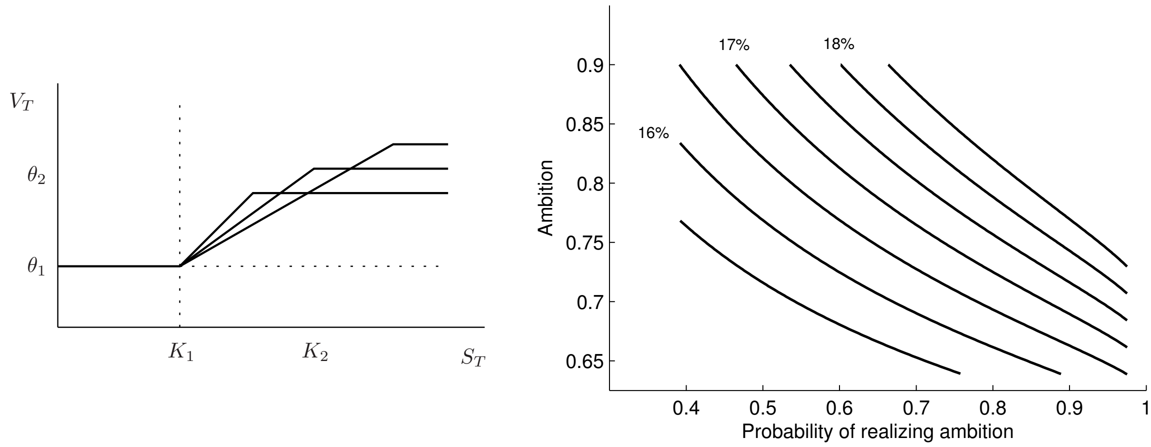

Figure 13: Trade-off between ambition and upper strike.

Floor and lower strike

Figure 14 illustrates the trade-off between the guaranteed level and the probability of ending up at the guarantee in terms of the option payoff scheme. In addition, the trade-off curves are depicted for a range of contribution rates, which are displayed next to the trade-off curves.

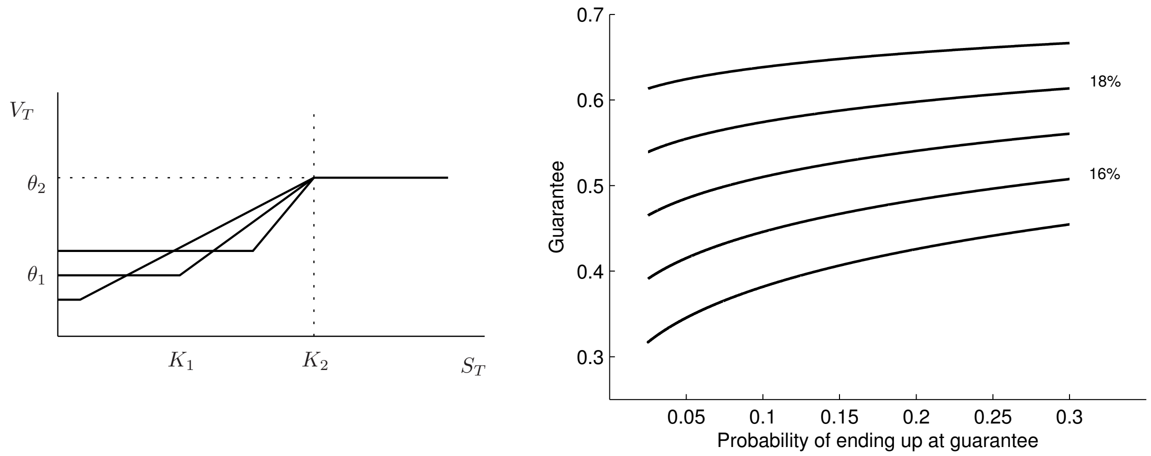

Figure 14: Trade-off between guarantee and lower strike.

Cap and lower strike

Figure 15 illustrates the trade-off between the ambition and the probability of ending up at the guarantee in terms of the option payoff scheme. In addition, the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves.

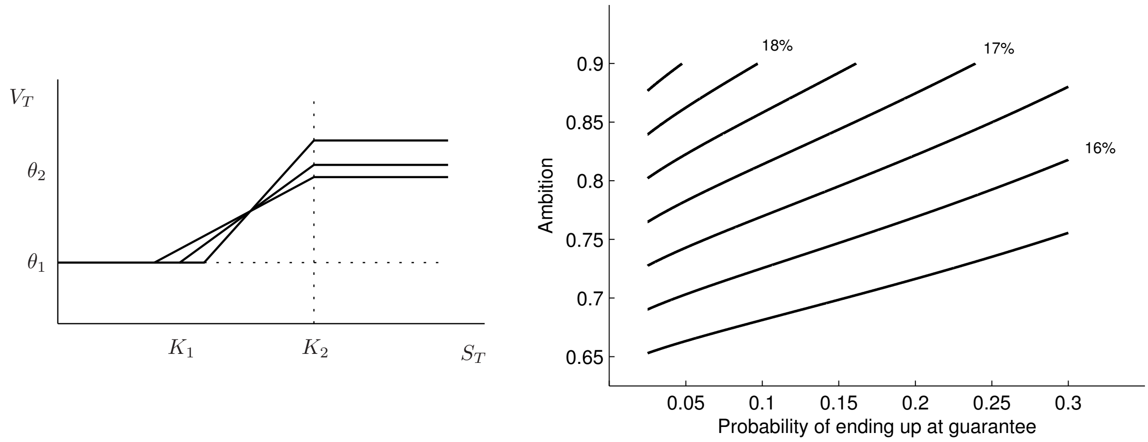

Figure 15: Trade-off between ambition and lower strike.

Floor and upper strike

Figure 16 illustrates the trade-off between the guarantee and the probability of realizing the ambition in terms of the option payoff scheme. In addition, the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves.

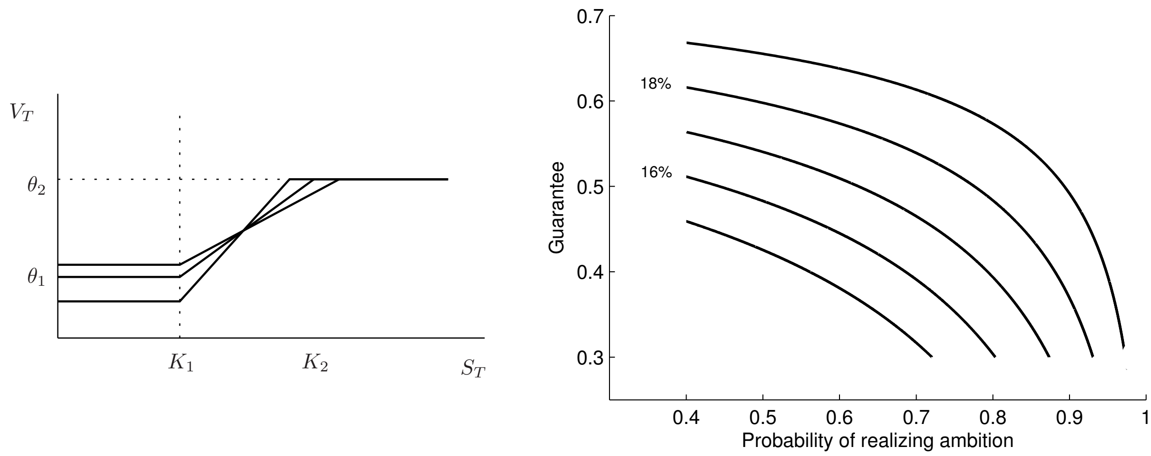

Figure 16: Trade-off between guarantee and upper strike.

B Comparison to myopic strategies

Performance in terms of desired and minimum required benefit

Figure 17 compares the performance of myopic schemes to the collar approach in terms of the desired and minimum required replacement rate that can be offered given predefined likelihoods.

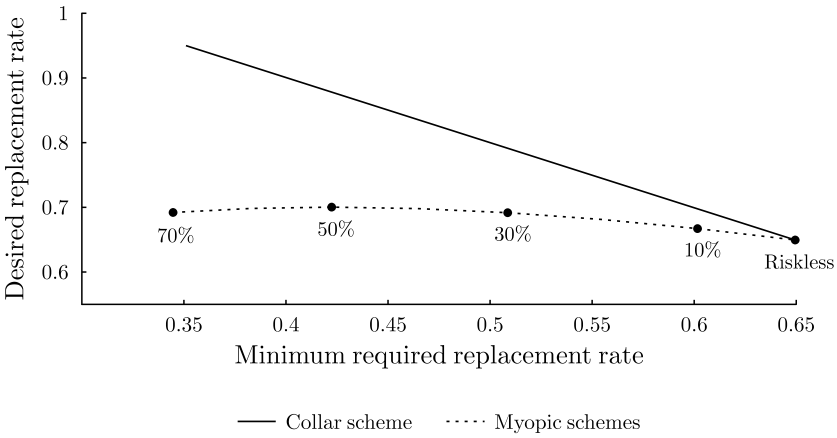

Figure 17: Trade-off between desired and minimum required replacement rate.

Performance in terms of likelihoods

Figure 18 compares the performance of myopic schemes to the collar approach in terms of the likelihoods that can be offered given predefined desired and minimum required replacement rates.

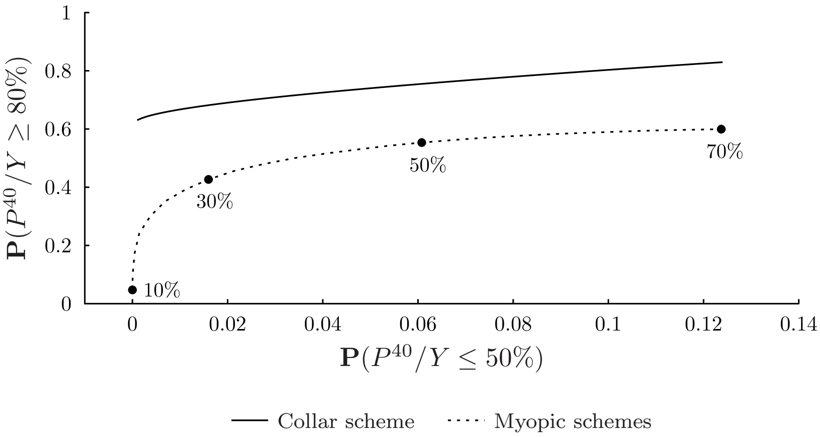

Figure 18: Probabilities of realizing desired and minimum required replacement rate.